Entanglement: Spooky (spoopy?) Action at a Distance

Note Title 2016-10-31

Q: What is entanglement?

$\rightsquigarrow$ correlations that are not realized by any classical system

$\rightsquigarrow$ nonlocal

ex an entangled state of two spins

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

e.g. pion decay, $\pi^0 \longrightarrow e^+ + e^-$

angular momentum: $|j,m\rangle = |0,0\rangle \approx \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$

- notice, if you hold spin #1 and measure in the z-basis, then based on your measurement result, you immediately know what the result of a measurement of spin #2 will be!

- alternatively, suppose you hold half of a pair of spins, and suppose that your task is to identify which joint state the pair is in:

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 \pm |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\uparrow\rangle_2 \pm |\downarrow\rangle_1|\downarrow\rangle_2\right)$$

- with only a single spin & no communication with your partner who holds the other spin, there's no way to identify the joint state

Apparently, entanglement is nonlocal

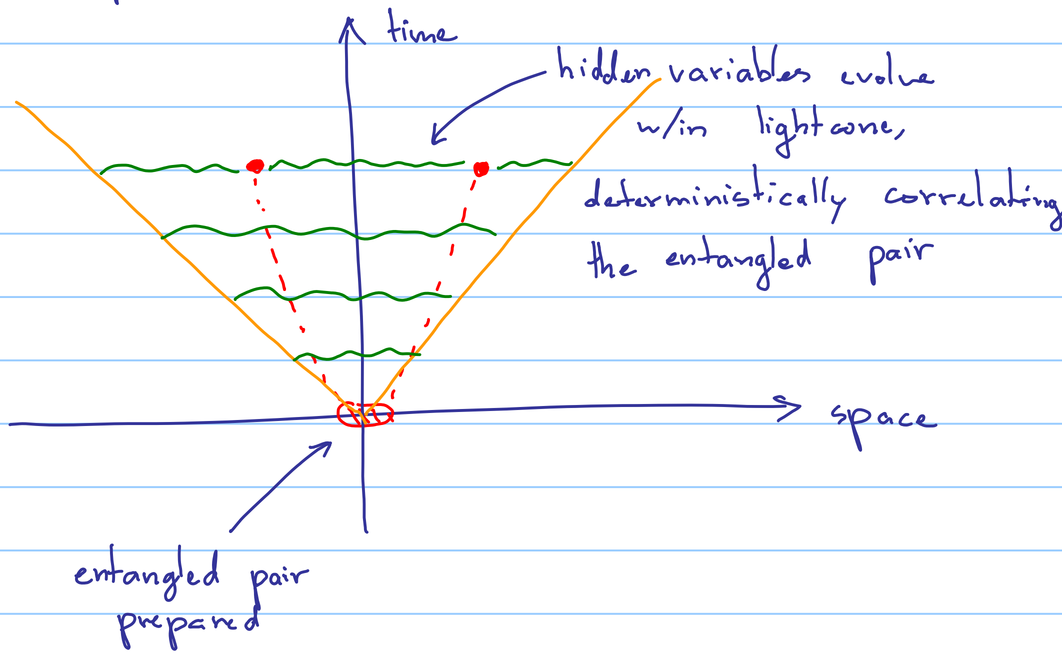
"Even at spacelike separations, actions on system 1 modify the description of system 2."

"Spooky Action at a Distance"

Einstein, Podolsky, & Rosen hated this! (EPR)

$\rightsquigarrow$ this led them to propose the following loophole:

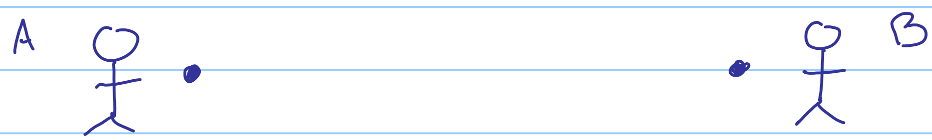
"Perhaps QM is an incomplete theory. In a complete theory, there are some local "hidden variables" that deterministically predict the outcomes of QM measurements."



Q: Can we test this?

$\to$ Yes! Bell experiments

Example CHSH experiment (variation on Bell's original proposal)



Alice: can measure a, a' Bob: can measure b, b'

- suppose outcomes of measurement are $\in \{+1, -1\}$
- consider if $a - a' = 0$ then $a + a' = \pm 2$ & vice-versa

$$\therefore \quad (a + a')b + (a - a')b' = \pm 2$$

• let $C = ab + a'b + ab' - a'b'$

$$\Rightarrow \boxed{|\langle C\rangle| \leq \langle |C| \rangle = 2}$$ CHSH inequality

• But now, suppose $a = \vec{\sigma}^A \cdot \hat{a}$ $\quad b = \vec{\sigma}^B \cdot \hat{b}$

$a' = \vec{\sigma}^A \cdot \hat{a}'$ $\quad b' = \vec{\sigma}^B \cdot \hat{b}'$

where $\hat{a}, \hat{a}', \hat{b}, \hat{b}'$ are unit vectors

& prepare the state $|\psi-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$

$$\vec{\sigma}\cdot\hat{a}\,|\uparrow\rangle = (a_x\sigma_x + a_y\sigma_y + a_z\sigma_z)|\uparrow\rangle$$
$$= a_x|\downarrow\rangle + ia_y|\downarrow\rangle + a_z|\uparrow\rangle$$

$$\vec{\sigma}\cdot\hat{a}\,|\downarrow\rangle = a_x|\uparrow\rangle - ia_y|\uparrow\rangle - a_z|\downarrow\rangle$$

$$\Downarrow$$

$$(\vec{\sigma}^A\cdot\hat{a})(\vec{\sigma}^B\cdot\hat{b})|\psi-\rangle = \frac{1}{\sqrt{2}}\left\{(\vec{\sigma}^A\cdot\hat{a}\,|\uparrow\rangle_A)(\vec{\sigma}^B\cdot\hat{b}\,|\downarrow\rangle_B) - (\vec{\sigma}^A\cdot\hat{a}\,|\downarrow\rangle_A)(\vec{\sigma}^B\cdot\hat{b}\,|\uparrow\rangle_B)\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{\left((a_x+ia_y)|\downarrow\rangle_A + a_z|\uparrow\rangle_A\right)\otimes\left((b_x-ib_y)|\uparrow\rangle_B - b_z|\downarrow\rangle_B\right)\right.$$
$$\left. - \left((a_x-ia_y)|\uparrow\rangle_A - a_z|\downarrow\rangle_A\right)\otimes\left((b_x+ib_y)|\downarrow\rangle_B + b_z|\uparrow\rangle_B\right)\right\}$$

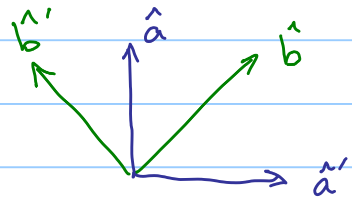
$$\Downarrow$$

$$\langle\psi-|(\vec{\sigma}^A\cdot\hat{a})(\vec{\sigma}^B\cdot\hat{b})|\psi-\rangle = \frac{1}{2}\left\{-(a_x+ia_y)(b_x-ib_y) - a_zb_z\right.$$
$$\left. - (a_x-ia_y)(b_x+ib_y) - a_zb_z\right\}$$

$$= \frac{1}{2}\left\{-a_xb_x - \cancel{ia_yb_x} + \cancel{ia_xb_y} - a_yb_y - a_zb_z\right.$$
$$\left. - a_xb_x + \cancel{ia_yb_x} - \cancel{ia_xb_y} - a_yb_y - a_zb_z\right\}$$

$$= -\hat{a}\cdot\hat{b}$$

So, choose $\hat{a}', \hat{b}, \hat{a}, \hat{b}'$ $45°$ apart



$$\hat{a}\cdot\hat{b} = \hat{a}'\cdot\hat{b} = \hat{a}\cdot\hat{b}' = \cos\frac{\pi}{4} = 1/\sqrt{2}$$
$$\hat{a}'\cdot\hat{b}' = \cos\frac{3\pi}{4} = -1/\sqrt{2}$$

$\rightarrow$ $\langle ab\rangle + \langle a'b\rangle + \langle ab'\rangle - \langle a'b'\rangle = 4\cdot(-1/\sqrt{2})$

So $|\langle ab\rangle + \langle ab'\rangle + \langle a'b\rangle - \langle a'b'\rangle| = 2\sqrt{2} > 2$!

Q: How did we evade the CHSH inequality?

$|\langle C\rangle| < 2$ supposes that we can simultaneously assign definite values to a, a', b, b'

$\rightarrow$ this is not the case in QM when observables don't commute!

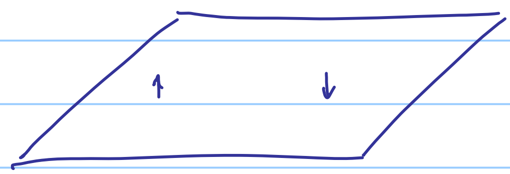
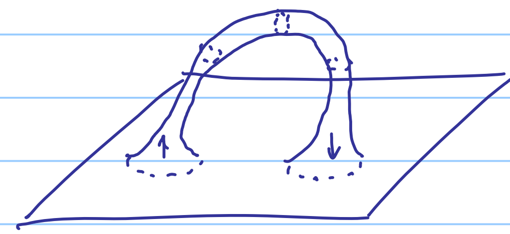
Q: Are there other loopholes / approaches to explaining QM nonlocality?

- ☆ nonlocal hidden variables (kind of defeats the purpose)
- ☆ contextuality
- ☆ ER = EPR new! exciting! crazy?

ER = EPR

→ "Einstein – Rosen" as in "Einstein-Rosen Bridge"
AKA wormholes

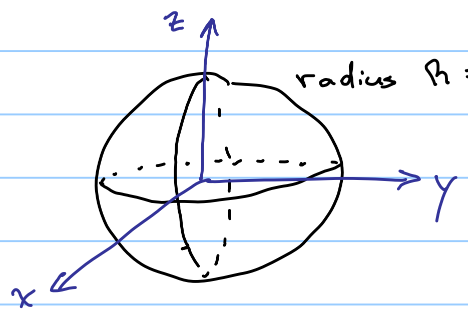
The basic idea:

what if  is actually 

- then, there's no problem with locality – entangled degrees of freedom are actually right next to each other!

- but, what the heck is a wormhole??
  Where do they come from? How do we make them?

Consider the Schwarzschild black hole



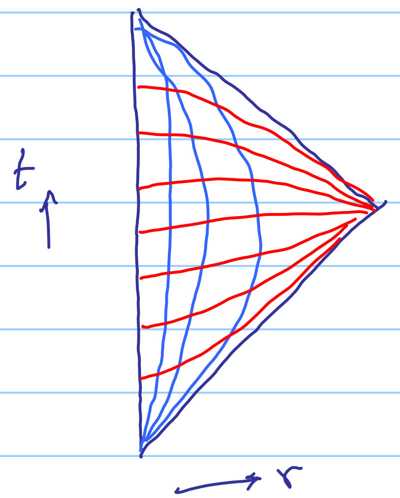
radius $R = \frac{2GM}{c^2}$

mass M

- in (t, x, y, z) coordinates, it's hard to draw the whole spacetime since it's infinite!
- idea: make a change of coordinates $(\tilde{t}, \tilde{r}, \theta, \varphi)$ so that the ranges of $\tilde{t}$, $\tilde{r}$ are finite
- also make sure that light rays are 45° lines

$\Rightarrow$ Penrose diagram

e.g. flat spacetime

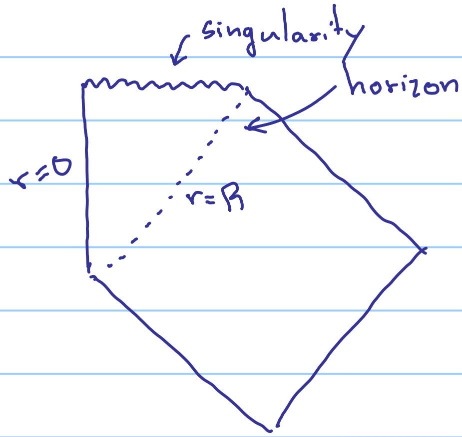


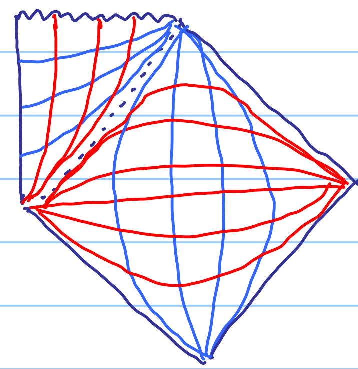
lines of constant r

lines of constant t

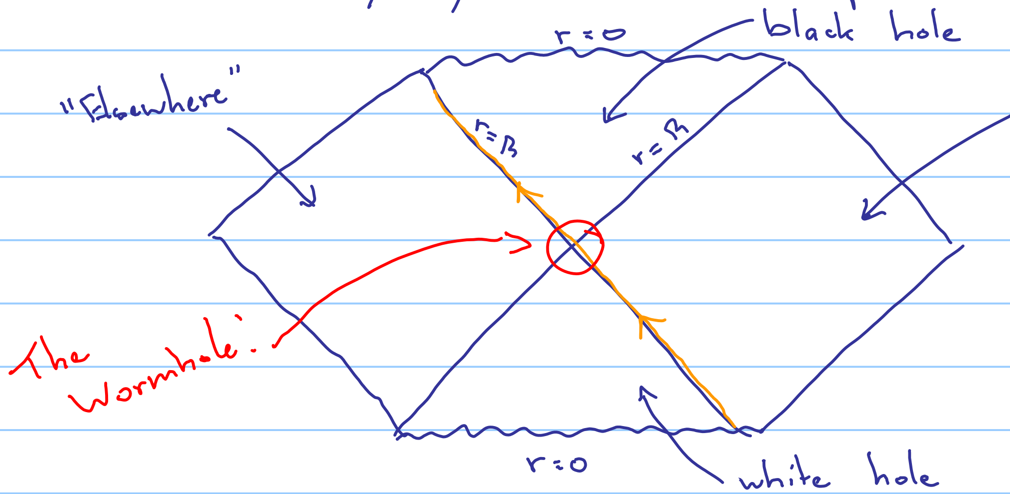
- each point is a sphere

eg. Schwarzschild



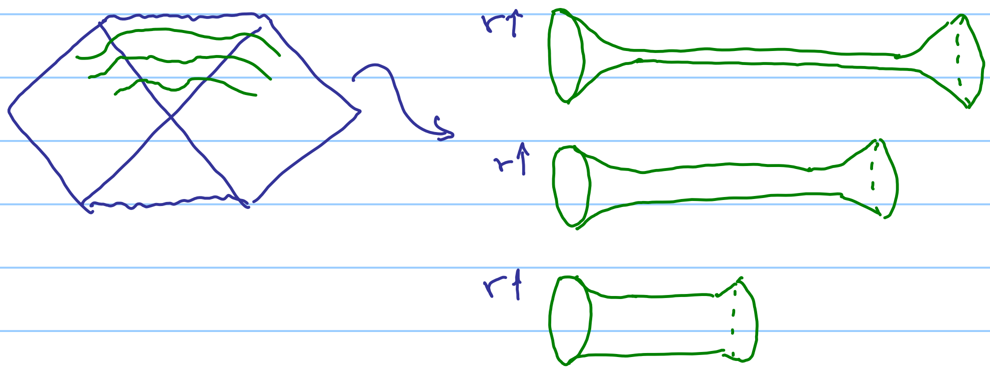


- then, if we "analytically extend" the spacetime



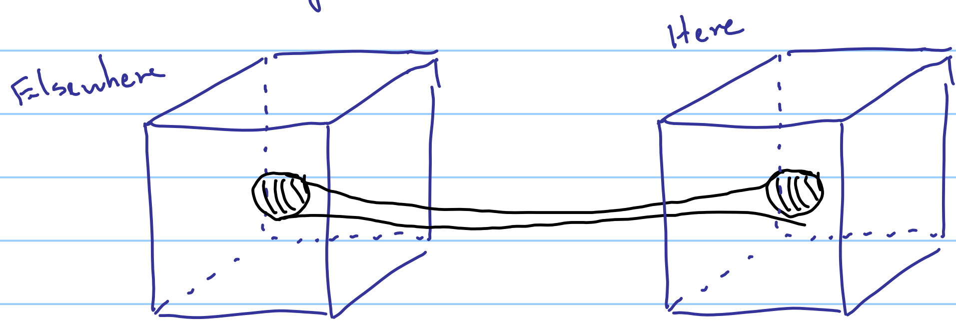
"Two-Sided Black Hole"

- the black hole acts like a "bridge" between Here and Elsewhere
- note that the wormhole is "not traversible"
  ~ you can't actually get to Elsewhere from Here

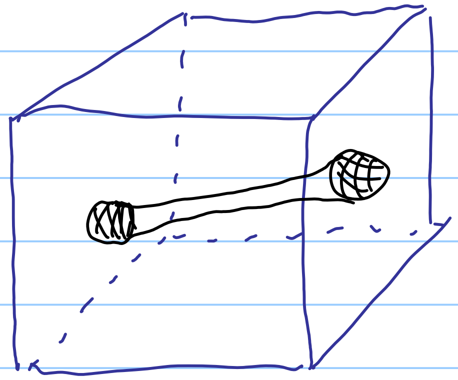


the throat pinches off!

- Now let's imagine that this



is actually



Now you have a single wormhole spacetime!

BUT What the heck has entanglement got to do with wormholes??

- ER = EPR is actually inspired from an observation in AdS/CFT, which is a conjectured duality between certain quantum field theories and certain gravitational systems

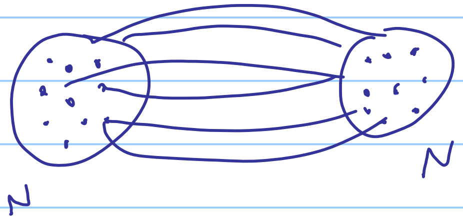
- in particular, certain entangled quantum states correspond to two-sided black holes w/ a wormhole

- ER = EPR is ① the wild suggestion that entanglement, at a macroscopic scale, is intimately important to wormhole structure, and

  ② the even wilder suggestion that at the microscopic level, entanglement produces some kind of "quantum wormhole," whatever that may be.

Now I don't claim to understand or be able to explain ER = EPR, but lets try to understand that entanglement itself is truly the important quantity for wormholes, and not the exact details of the state.

Model for a 2-sided BH: N pairs of entangled spins:



Q: What state should we use?

- it should be entangled

eg. $|\Psi\rangle = |\psi^-\rangle^{\otimes N}$ N copies of $|\psi^-\rangle$

- but, this state is pretty special, not so great
- it turns out that a random state is actually pretty close to maximally entangled when split in 2

Let $|\Psi\rangle = \sum_x c_{x_1 \ldots x_n} |x_1 \ldots x_n\rangle$ ← each $x_j \in \{0,1\}$

$x \equiv x_1 \cdots x_n$ n bit string

$c_{x_1 \ldots x_n} \equiv c_x = r_x e^{i\phi_x}$, $r = \text{rand}(0,1)$

$\phi = \text{rand}(0, 2\pi)$

(of course subject to $\sum |c_x|^2 = 1$)

Claim: entanglement is important, not details of the state itself, e.g. purity

observe: sps. we remove one pair of spins...

- N is very large → shouldn't affect the wormhole
- does entanglement change? Not much!
- purity?

ex try tracing out one spin

$$\rho \longrightarrow \text{Tr}_1 \rho = \langle 0_1 | \Psi \rangle\langle \Psi | 0_1 \rangle + \langle 1_1 | \Psi \rangle\langle \Psi | 1_1 \rangle$$

$$\langle 0_1 | \Psi \rangle = \sum_{x_2 \cdots x_n} c_{0 x_2 \cdots x_n} |x_2 \cdots x_n\rangle \equiv \sum_x c_{0x} |x\rangle$$

Similarly, $\langle 1_1 | \Psi \rangle = \sum_x c_{1x} |x\rangle$

So $\mathrm{Tr}_1 \rho = \left(\sum_x c_{0x} |x\rangle \right) \left(\sum_y c_{0y}^* \langle y| \right) + \left(\sum_x c_{1x} |x\rangle \right) \left(\sum_y c_{1y}^* \langle y| \right)$

$$= \sum_{x,y} \left(c_{0x} c_{0y}^* + c_{1x} c_{1y}^* \right) |x\rangle\langle y| \equiv \tilde{\rho}$$

Q: How to measure purity?

$\rightarrow$ look at $\mathrm{Tr}\,\tilde{\rho}^2$ ... $\mathrm{Tr}\,\tilde{\rho}^2 = 1$ if pure

$$\tilde{\rho}^2 = \sum_{x,y} \sum_{x',y'} \left(c_{0x} c_{0y}^* + c_{1x} c_{1y}^* \right) \left(c_{0x'} c_{0y'}^* + c_{1x'} c_{1y'}^* \right) |x\rangle \overbrace{\langle y|x'\rangle}^{\delta_{yx'}} \langle y'|$$

$$= \sum_{xay} \left(c_{0x} c_{0a}^* + c_{1x} c_{1a}^* \right) \left(c_{0a} c_{0y}^* + c_{1a} c_{1y}^* \right) |x\rangle\langle y|$$

$$\mathrm{Tr}\,\tilde{\rho}^2 = \sum_{xa} \left(c_{0x} c_{0a}^* + c_{1x} c_{1a}^* \right) \left(c_{0a} c_{0x}^* + c_{1a} c_{1x}^* \right)$$

$$= \sum_{xa} |c_{0x}|^2 |c_{0a}|^2 + |c_{1x}|^2 |c_{1a}|^2 + c_{0x} c_{0a}^* c_{1a} c_{1x}^* + c_{1x} c_{1a}^* c_{0a} c_{0x}^*$$

$$= \left(\sum_x |c_{0x}|^2 \right) \left(\sum_a |c_{0a}|^2 \right) + \left(\sum_x |c_{1x}|^2 \right) \left(\sum_a |c_{1a}|^2 \right)$$

$$+ 2\,\mathrm{Re} \underbrace{\left(\sum_x c_{0x} c_{1x}^* \right) \left(\sum_a c_{0a}^* c_{1a} \right)}_{= \left| \sum_x c_{0x} c_{1x}^* \right|^2}$$

Now, the c's are random, so expect $|c_{0x}|^2 \sim |c_{1x}|^2 \sim \frac{1}{2^n}$

$$\therefore \mathrm{Tr}\,\tilde{\rho}^2 \sim \left(\frac{2^{n-1}}{2^n} \right) \left(\frac{2^{n-1}}{2^n} \right) + \left(\frac{2^{n-1}}{2^n} \right) \left(\frac{2^{n-1}}{2^n} \right) + 2 \left| \sum_x c_{0x} c_{1x}^* \right|^2$$

treat like a random phase

$$2\left|\sum_x c_{0x} c_{1x}^*\right|^2 \sim 2 \cdot \left|\frac{1}{2^n} \sum_x e^{i\phi(x)}\right|^2$$

$$= \frac{1}{2^{2n-1}} \left|\sum_x e^{i\phi(x)}\right|^2$$

Q: if each $\phi(x) = \text{rand}(0, 2\pi)$, what is $\left\langle \left|\sum_x e^{i\phi(x)}\right|^2 \right\rangle$?

ans: $\left\langle \left|\sum_x e^{i\phi(x)}\right|^2 \right\rangle = 2^{n-1}$

$\therefore \quad \mathrm{Tr}\,\tilde{\rho}^2 \sim \frac{1}{4} + \frac{1}{4} + \frac{1}{2^n} \longrightarrow \frac{1}{2}$ if n is large

$\Rightarrow$ removing even a single spin (e.g. via Hawking evaporation) totally destroys purity!