Problem 1

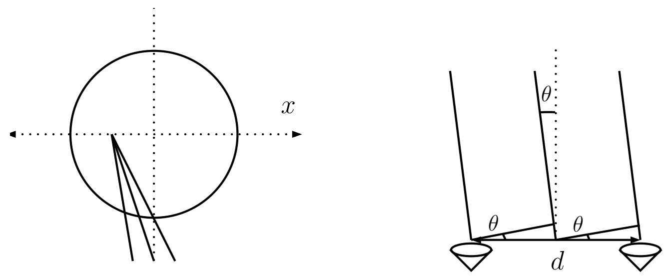

Figure 1: A point on Sirius emitting three light rays, $R = 8.5\ ly$ away, and the detectors on Earth (nothing drawn to scale, neither absolute nor relative).

Referring to Fig. 1, the difference in path lengths between detectors A and B is $\delta = d\sin\theta \approx d\theta$, and the maximum value that θ takes is $\theta \approx \tan\theta = r/R$. Therefore,

$$\phi \in \left[-2\pi\frac{\delta}{\lambda}, 2\pi\frac{\delta}{\lambda}\right] \approx \left[-2\pi\frac{rd}{\lambda R}, 2\pi\frac{rd}{\lambda R}\right] .$$

According to the Wien law, the peak wavelength at $T = 10000\ K$ is

$$\lambda = \frac{3\times 10^{-3}\ m\cdot K}{T} = 3\times 10^{-7}\ m\,.$$

Plugging in the numbers,

$$2\pi\frac{rd}{\lambda R} = 2\pi\frac{rd}{3\times 10^{-7}\cdot 8.5\cdot \pi\times 10^{7}\cdot 3\times 10^{8}} \approx \frac{rd}{4}\times 10^{-9}\ m^{-2}\,,$$

so ϕ ranges between plus and minus that.

Problem 2

The phase $\phi(x)$ is just given by

$$\phi(x) = 2\pi\frac{xd}{\lambda R}\,.$$

The correlator is hence

$$C_{AB} = f_0^2\int_{-r}^{r} dx\ e^{-2x^2/r^2}\cos\left(2\cdot 2\pi\frac{xd}{\lambda R}\right) \approx f_0^2\int_{-\infty}^{\infty} dx\ e^{-2x^2/r^2}\cos\left(4\pi\frac{xd}{\lambda R}\right)\,.$$

Then, it's just algebra. Let $\alpha = 2/r^2$, $\beta = 4\pi d/\lambda R$.

$$\begin{aligned}
C_{AB} &= f_0^2 \int_{-\infty}^{\infty} dx\; e^{-\alpha x^2} \cos(\beta x) \\
&= f_0^2 \int_{-\infty}^{\infty} dx\; e^{-\alpha x^2} \frac{1}{2} \left(e^{i\beta x} + e^{-i\beta x}\right) \\
&= \frac{f_0^2}{2} \int_{-\infty}^{\infty} dx\; \left(e^{-\alpha(x-i\beta/2\alpha)^2 - \beta^2/4\alpha} + e^{-\alpha(x+i\beta/2\alpha)^2 - \beta^2/4\alpha}\right) \\
&= \frac{f_0^2}{2} e^{-\beta^2/4\alpha} \left(\sqrt{\frac{\pi}{\alpha}} + \sqrt{\frac{\pi}{\alpha}}\right) \\
&= f_0^2\; e^{-\beta^2/4\alpha} \sqrt{\frac{\pi}{\alpha}}
\end{aligned}$$

Problem 3

First normalize the correlator, i.e., $C_{AB}(d=0) = f_0^2\sqrt{\pi/\alpha}$, so

$$\Gamma(d) = e^{-\beta^2/4\alpha} = \exp\left(-\frac{1}{2}\left(\frac{2\pi r d}{\lambda R}\right)^2\right) \tag{1}$$

Plugging in $r/R = 2\ mas$, or about $10^{-8}\ rad$, produces a pretty good fit (taking $\lambda = 3 \times 10^{-7}\ m$ as before).