## Hanbury-Brown - Twiss and Sirius

When two light receivers are used to monitor the light of a far away source, Hanbury-Brown and Twiss showed that if there is a phase difference  $\phi$  for a single beam of light arriving at the two receivers then the product of the observed signal is:

$$C_{AB} = \langle (i_A - \langle i_A \rangle)(i_B - \langle i_B \rangle) \rangle = \frac{I^2}{8} \cos 2\phi \tag{1}$$

where the brackets indicate time averaging, and the individual intensities observed in the detectors 1 and 2.

In their experiment, HB&T trained their detectors on Sirius, R = 8.5 ly away, and measured the correlation  $C_{12}(d)$  as a function of distance between the detectors d(see figure). The graph given actually plots the result for

$$\Gamma(d) = \frac{C_{AB}(d)}{C_{AB}(d=0)} \tag{2}$$

which is the reduced correlation between the two detectors, normalized by the maximum correlations (when they are right next to eachother).

Assume that  $\theta = 0$  is chosen as the center of the star.

- 1. What is the range of phase differences  $\phi$  that will be observed in the detectors at baseline d, if the radius of Sirius is r? Assume that the relevant wavelength is the Sirius black-body maximum corresponding to T = 10000K.
- 2. Assume the star intensity varies along the star, say as:

$$f(x) \sim f_0 e^{-x^2/r^2} \tag{3}$$

with x measuring the distance of a point on the star from its center along a direction parallel to d - the line between the two detectors. We expect that the correlations observed would be:

$$C_{12}(d) = f_0^2 \int dx e^{-2x^2/r^2} \cos 2\phi(x)$$
(4)

First, find the relationship between  $\phi(x)$  and x. Hint: simply replace r with x in the expression for the previous item. Next, carry out the integral.

Instructions for the integral:

$$e^{-ax^2 + ikx} = e^{-a(x+ik/2a)^2 - k^2/4a}$$
(5)

and:

$$\int_{-\infty}^{\infty} dx e^{a(x-x_0)^2} = \sqrt{\pi/a}.$$
(6)

3. Which r/R fits the data best?



FIG. 1. (a) The two detectors, A and B, are used to observe the star Sirius. First choosing the best  $\theta$  for the observation, and then measuring the correlations as a function of d, HBT determined  $2\delta\theta$  - the angular width of Sirius. (b) The resulting correlation graph. Can you reproduce it?