LOOKING AT BLACK HOLES WITH THE EVENT HORIZON TELESCOPE

Black holes are ubiquitous in modern physics. They play an important role in theoretical physics because they are some of the few systems for which both quantum effects and gravitational effects are important. They are extremely important for astrophysics, too. Black holes are a terminal stage in the stellar life cycle, and we have a huge amount of indirect evidence that they exist and are important for the dynamics of star systems and galaxies. Most recently, we have metaphorically "seen" pairs of black holes as they merge by detecting the gravitational waves given off during their final inspiral. However, soon we will have literally seen black holes as well!

This will be the case if everything goes according to plan for the Event Horizon Telescope (EHT), which is a large international collaboration that has linked radio observatories around the world to observe Sagittarius A*, the black hole at the centre of our galaxy. In doing so, they effectively turned the entire Earth into a radio telescope. But, how do you use an Earth-sized array of radio telescopes to take a visual picture of a black hole? The short answer is interferometry. For the long answer, read on...

A. Warmup: How big does the telescope have to be?

We've seen interferometry in astronomy before—remember Hanbury-Brown and Twiss' experiment? Before jumping back into the physics of interferometry, however, let's get a sense for the scales of the problem.

So, there's a black hole at the centre of the galaxy called Sagittarius A* (Sgr A*), and its mass is about 4 million solar masses, so

$$M_{A*} = 8 \times 10^{36} \text{ kg}. \tag{1}$$

The radius of a black hole—the Schwarzschild radius—is given by

$$r_s = \frac{2GM}{c^2}, \tag{2}$$

so the radius of Sgr A* is about

$$r_{A*} = 1.2 \times 10^{10} \text{ m}. \tag{3}$$

We are about 8000 parsecs away from the centre of the galaxy, so, in metres, let's take the distance to Sgr A* to be

$$R = 2.5 \times 10^{20} \text{ m}. \tag{4}$$

Now suppose that you wanted to build a telescope to observe Sgr A*. How big would it have to be? Be it an optical telescope or an interferometric one, up to some $O(1)$ constants, the angular resolution of the telescope is

$$\theta \sim \frac{\lambda}{D}, \tag{5}$$

where λ is the wavelength of the light being observed and D is the size of the telescope's aperture (or rather, the telescope's baseline for an interferometric telescope). The angular size that Sgr A* occupies in the sky is

$$\frac{\theta}{2} \approx \tan\frac{\theta}{2} = \frac{r_{A*}}{R}, \tag{6}$$

and so we had better have that

$$D \sim \frac{\lambda}{\theta} \approx \frac{\lambda R}{2 r_{A*}}. \tag{7}$$

Taking $\lambda \sim 1.3$ mm, which is the peak frequency of the radio band in which the EHT operates, the telescope size comes out to

$$D \sim 1.4 \times 10^7 \text{ m}, \tag{8}$$

or 14,000 km. The radius of the Earth is $r_E = 6,500$ km, so if you double that, you find that the diameter of the Earth seems to be a big enough baseline for directly observing Sgr A*!

If you instead take $\lambda \sim 500$ nm for visible light, you find that D is only 5 km or so. That seems great at first, but unfortunately we're not going to build a 5-km large lens any time soon. So, for the time being, radio astronomy is the way to go.

B. How to take a picture using interferometers

The EHT consists of several radio telescopes located all around the Earth. Each pair of radio telescopes is therefore a radio interferometer, and so the basic setup is actually very similar to Hanbury-Brown and Twiss' single interferometer. Hanbury-Brown and Twiss were only concerned with measuring the size of a star, however. How do you extract a photograph from interferometric data?

To answer this question, let's review how a pair of radio detectors functions as an interferometer. Consider two radio telescopes on Earth that we label by A and B, and suppose that they are trained on some distant source. From the ground, the source looks like a 2D image on the sky. The source looks like a flat image since it takes up only a very small portion of the visible sky, so let's draw two Cartesian coordinate axes in the plane of the sky and centred somewhere in the source, as shown in Fig. 1.

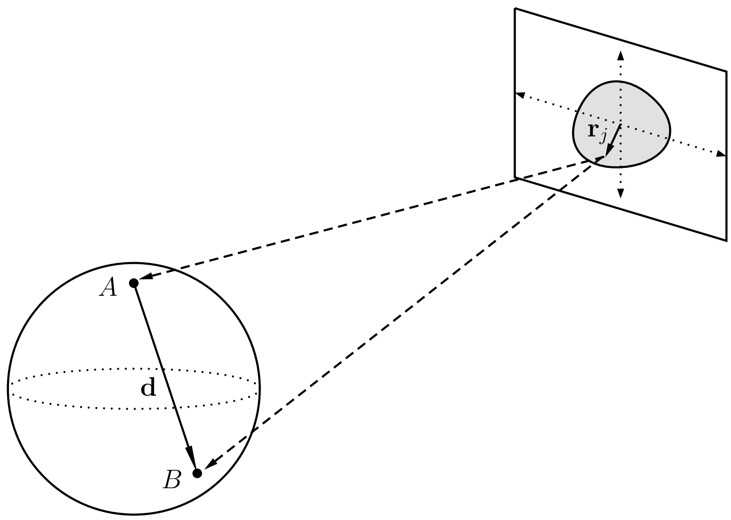

FIG. 1. Two radio detectors on Earth, labelled A and B, that could be a part of the EHT. They are separated by a baseline vector $\mathbf{d}$. They are used to image an object in the plane of the sky. The dashed arrows show the path taken by light coming from the point $\mathbf{r}_j$ on the emitting source that impinges on the two detectors.

Now consider the light emitted by two points on the source at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ in this coordinate system. Denote the intensity of the light measured by detector A coming from the points on the source at $\mathbf{r}_1$ and $\mathbf{r}_2$ by $I^A(t)$, and similarly for detector B. We can write these intensities as follows:

$$I^A(t) = I_0(\mathbf{r}_1)\cos^2(\omega_1 t + \phi_1^A) + I_0(\mathbf{r}_2)\cos^2(\omega_2 t + \phi_2^A) \tag{9}$$

$$I^B(t) = I_0(\mathbf{r}_1)\cos^2(\omega_1 t + \phi_1^B) + I_0(\mathbf{r}_2)\cos^2(\omega_2 t + \phi_2^B) \tag{10}$$

In other words, we are supposing that the light coming from $\mathbf{r}_1$ and $\mathbf{r}_2$ has frequencies ω_1 and ω_2 respectively, and that it arrives at detector A with a phase shifts ϕ_1^A, ϕ_2^A; and at detector B with a phase shifts ϕ_1^B, ϕ_2^B. Note that the intensity profile $I_0(\mathbf{r})$ is what we want to measure. This function is essentially a monochromatic picture of the emitting source!

The two detectors work as an interferometer by multiplying the two signals together and averaging over time. As before, begin by exploiting some trigonometric identities to write

$$I^A(t) = \tfrac{1}{2}I_0(\mathbf{r}_1)\left(1 + \cos(2\omega_1 t + 2\phi_1^A)\right) + \tfrac{1}{2}I_0(\mathbf{r}_2)\left(1 + \cos(2\omega_2 t + 2\phi_2^A)\right) \tag{11}$$

$$I^B(t) = \tfrac{1}{2}I_0(\mathbf{r}_1)\left(1 + \cos(2\omega_1 t + 2\phi_1^B)\right) + \tfrac{1}{2}I_0(\mathbf{r}_2)\left(1 + \cos(2\omega_2 t + 2\phi_2^B)\right)\ . \tag{12}$$

Let's again subtract off the averages $\langle I^A\rangle = \langle I^B\rangle = \frac{1}{2}I_0(\mathbf{r}_1) + \frac{1}{2}I_0(\mathbf{r}_2)$, and to change things up a bit, this time let's write the intensities using complex exponentials:

$$I^A(t) - \langle I^A\rangle = \tfrac{1}{4}I_0(\mathbf{r}_1)\left(e^{2i(\omega_1 t+\phi_1^A)} + e^{-2i(\omega_1 t+\phi_1^A)}\right) + \tfrac{1}{4}I_0(\mathbf{r}_2)\left(e^{2i(\omega_2 t+\phi_2^A)} + e^{-2i(\omega_2 t+\phi_2^A)}\right) \tag{13}$$

$$I^B(t) - \langle I^B\rangle = \tfrac{1}{4}I_0(\mathbf{r}_1)\left(e^{2i(\omega_1 t+\phi_1^B)} + e^{-2i(\omega_1 t+\phi_1^B)}\right) + \tfrac{1}{4}I_0(\mathbf{r}_2)\left(e^{2i(\omega_2 t+\phi_2^B)} + e^{-2i(\omega_2 t+\phi_2^B)}\right) \tag{14}$$

The correlation function of the two detectors is then

$$C^{AB} = \langle (I^A(t) - \langle I^A \rangle)(I^B(t) - \langle I^B \rangle) \rangle, \tag{15}$$

and this is what we can actually measure by measuring the intensities $I^A(t)$ and $I^B(t)$ over time.

The only terms from the product $(I^A(t) - \langle I^A \rangle)(I^B(t) - \langle I^B \rangle)$ that survive the time averaging in C^{AB} are those that have no time dependence, and so

$$C^{AB} = \tfrac{1}{16} I_0(\mathbf{r}_1)^2 \left(e^{2i(\phi_1^A - \phi_1^B)} + e^{-2i(\phi_1^A - \phi_1^B)} \right) + \tfrac{1}{16} I_0(\mathbf{r}_2)^2 \left(e^{2i(\phi_2^A - \phi_2^B)} + e^{-2i(\phi_2^A - \phi_2^B)} \right) . \tag{16}$$

Alternatively, the complexified correlation function is

$$\tilde{C}^{AB} = \tfrac{1}{8} I_0(\mathbf{r}_1)^2 e^{2i(\phi_1^A - \phi_1^B)} + \tfrac{1}{8} I_0(\mathbf{r}_2)^2 e^{2i(\phi_2^A - \phi_2^B)}, \tag{17}$$

where we simply understand that $C^{AB} = \text{Re}\ \tilde{C}^{AB}$. Now that we've done the calculation for 2 points on the source, it's not too much of a stretch to see that for n points on the source at positions $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$, the correlator is given by

$$\tilde{C}^{AB} = \frac{1}{8} \sum_{j=1}^{n} I_0(\mathbf{r}_j)^2 e^{2i\Delta\phi_j}, \tag{18}$$

where $\Delta\phi_j \equiv \phi_j^A - \phi_j^B$. Moreover, we can figure out what this phase difference is in terms of geometric parameters.

Formally, the phase difference is given by

$$\Delta\phi_j = 2\pi \frac{\delta_j}{\lambda}, \tag{19}$$

where δ_j is the difference in path lengths from the point on the source at $\mathbf{r}_j$ to the detectors at A and at B. Remember how it was in one fewer dimension in the Hanbury-Brown and Twiss analysis: When the source is one-dimensional and the source and baseline are in the same plane, this path difference was given by

$$\delta_j = d \sin\theta \approx d \tan\theta = d \frac{r_j}{R}, \tag{20}$$

where d was the distance between the two detectors (Fig. 2). (Recall that we can always make the baseline effectively parallel to the source by adding a delay line to one of the two detectors to artificially increase the path length.)

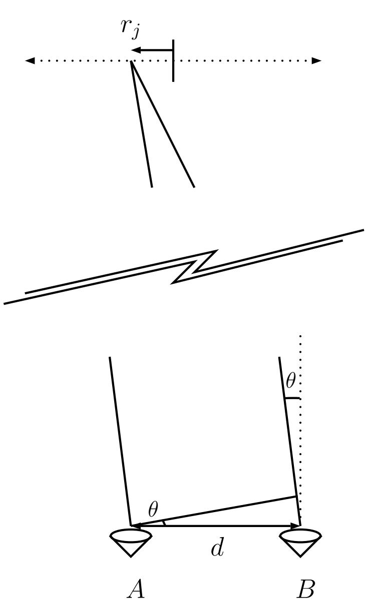

FIG. 2. The case when the light source is 1D and is coplanar with and parallel to the detector baseline. The figure is not to scale

In the case at hand, the baseline is now a vector $\mathbf{d}$ that we draw from A to B (which, by being crafty with delay lines, we can adjust to be parallel to the 2D plane in which $\mathbf{r}_j$ lies). Clearly we have to reproduce the same result when the baseline and $\mathbf{r}_j$ are in the same plane. Also note that when $\mathbf{d}$ and $\mathbf{r}_j$ are perpendicular, then the difference in path lengths vanishes. Therefore, it perhaps isn't much of a surprise that the correct generalization is

$$\delta_j = \frac{\mathbf{d} \cdot \mathbf{r}_j}{R}. \tag{21}$$

Putting it all together, we finally arrive at the following expression for the correlator:

$$\tilde{C}^{AB}(\mathbf{d}) = \frac{1}{8} \sum_{j=1}^{n} I_0(\mathbf{r}_j)^2 \exp\left\{4\pi i \frac{\mathbf{d} \cdot \mathbf{r}_j}{\lambda R}\right\} \tag{22}$$

Now we see what $\tilde{C}^{AB}$ really is: up to some prefactors and constants, it's the discrete Fourier transform of $I_0(\mathbf{r})^2$, where the Fourier partner variable for $\mathbf{r}$ is $\mathbf{d}$. Therefore, if you can measure $\tilde{C}^{AB}(\mathbf{d})$, then you can invert the Fourier transform to get $I_0(\mathbf{r})$.

This is essentially how the EHT works. By taking pairs of radio telescopes around the world, and also just by sitting around and letting the Earth rotate, you get to measure $\tilde{C}^{AB}$ for a large set of different baseline vectors $\mathbf{d}$, from which you can reconstruct an image of Sgr A*.