Lecture 4 - Applications to Holography

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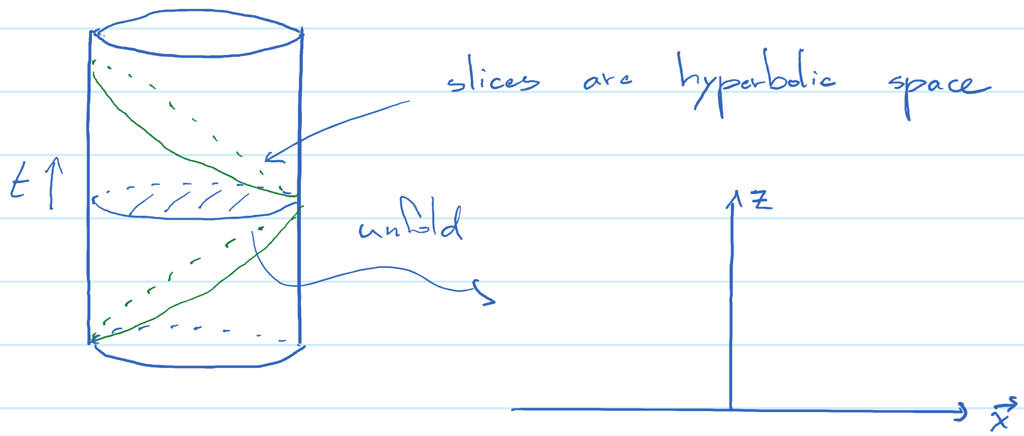
AdS/CFT Conjecture:

- Certain Conformal Field Theories (w/o gravity) in d dims are exactly equivalent to quantum theories of asymptotically Anti-de Sitter spacetimes in $d+1$ dimensions
- In the right limit, certain CFT states are in exact corresp. w/ certain fixed asymp. AdS spacetimes.

ex $|0\rangle$ of CFT_d $\leftrightarrow$ pure AdS_{d+1} const. $-$ive curvature



Poincaré patch coords: $ds^2 = \frac{\ell^2}{z^2}\left(-dt^2 + dz^2 + dx_i dx^i\right)$
(not global, but illustrative)

↖ fixed coord distances near boundary $z=0$ get longer

cc: $\Lambda = -\frac{d(d-1)}{2\ell^2}$

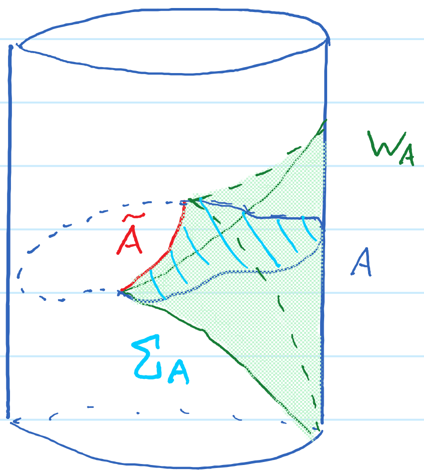
- Think of CFT_d as living on the boundary

Is it a general theory of QG? No
Does it teach us something about QG? Hopefully!

QI Connection:

info quantities in CFT $\longleftrightarrow$ geometric quantities in AdS

Ryu-Takayanagi; Hubeny-Rangamani-Takayanagi



$$S(\rho_A) = \min_{\tilde{A}\sim A} \operatorname{ext} \frac{|\tilde{A}|}{4G_N} \quad — (*)$$

ρ_A: CFT reduced on bdy subregion A
$\tilde{A}$: extremal surface homologous to A

extremal: $\delta|\tilde{A}| = 0$ $\quad (\Leftrightarrow \theta_k, \theta_\ell = 0)$
homologous: can be smoothly deformed to A

note: (*) is covariant version due to HRT. Original RT formula for static case is $S(\rho_A) = \min_{\tilde{A}\sim A} \frac{|\tilde{A}|}{4G_N}$

Bulk Reconstruction

- True duality ⇒ any bulk quantity encoded in bdy CFT
- Q: bulk operators?

HKLL (Hamilton, Kabat, Lifschytz, Lowe)
- earliest formulation
- free scalar ϕ on AdS_3
- based on "extrapolate dictionary"

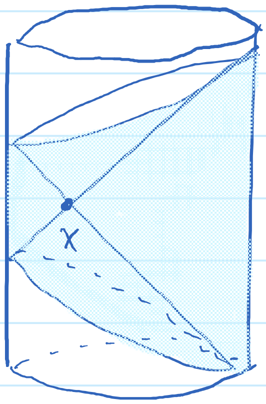
$$\lim_{r\to\infty} r^\Delta \phi(r,t,\Omega) = \mathcal{O}(t,\Omega) \quad — (1)$$

↖ CFT primary w/ scaling dim Δ

- basic strat: find CFT op that satisfies (1) as well as bulk EOM

$$(\Box - m^2)\tilde{\phi} = 0$$

↙ a CFT op!



$$\tilde{\phi}(x) = \int dY \, K(x,Y) \, \mathcal{O}(Y)$$

- supp. on bdy points spacelike-sep from x
- $K \equiv$ "smearing function"
  (in terms of ϕ mode functions, ...)

- as desired, $\langle \phi(x_1) \phi(x_2) \cdots \rangle_{AdS} = \langle \tilde{\phi}(x_1) \tilde{\phi}(x_2) \cdots \rangle_{CFT}$

Q: more "efficient" rep.?
$\rightsquigarrow$ eg. could propagate $K(x,Y)$ onto a single bdy Cauchy slice
Q: most efficient rep?

Entanglement Wedge Reconstruction

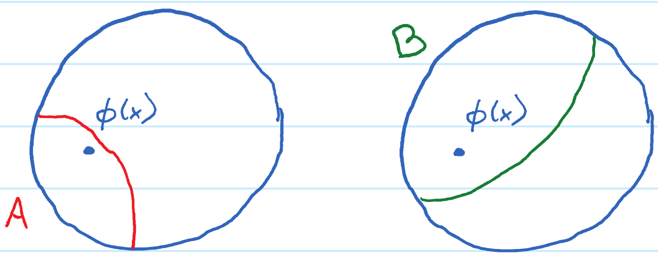
Def^n Given bdy subregion A w/ HRT surf. $\tilde{A}$, the entanglement wedge of A is the bulk domain of dependence of any spacelike Σ_A such that $\partial \Sigma_A = A \cup \tilde{A}$

// domain of dep. of Σ_A = $\{ p \mid$ any causal curve $\ni p$ intersects $\Sigma_A \}$

EWR: any bulk op. in W_A can be represented as a CFT op. w/ support on A

Note: this is what is meant by subregion-subregion duality
$\rightarrow$ $A \in$ CFT $\Leftrightarrow$ W_A in bulk

Note: representation is redundant... what gives?



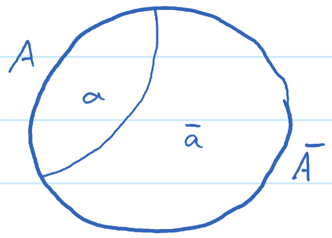
$\mathcal{O}_A$ or $\mathcal{O}_B$??

Intuit: bulk reconstruction is a QECC that protects against deletion of portions of the bdy

$\rightarrow$ bulk encoded redundantly, nonlocally in CFT!

Bulk Reconstruction via Universal Recovery Channels

[Cotler, Hayden, Penington, Salton, Swingle, Walter ; 2019]



$\mathcal{H}_{CFT} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$

$\mathcal{H}_{bulk} = \mathcal{H}_a \otimes \mathcal{H}_{\bar{a}}$

(can make all args. w/ op alg. but for convenience here assume factorization)

- let $\mathcal{H}_{code} \subset \mathcal{H}_{bulk}$: finite collection of states that have same bulk geom in neighbourhood of a to $O(1/N)$
- holography: $\exists$ isometry $J : \mathcal{H}_{code} \hookrightarrow \mathcal{H}_{CFT}$

// again, can replace J w/ arb. channel, but for convenience

Goal: given ϕ_a w/ supp. on a, find O_A w/ supp. on A s.t.

$$\left| \langle O_A \rangle_{J\rho J^\dagger} - \langle \phi_a \rangle_\rho \right| \leq \delta \, \| \phi_a \| \quad \forall \; \rho \in S(\mathcal{H}_{code})$$

Crucial ingredient: JLMS [Jafferis, Lewkowycz, Maldacena, Suh]

$$\left| D(\rho_A \| \sigma_A) - D(\rho_a \| \sigma_a) \right| \leq O(1/N) \quad \forall \; \rho, \sigma \in \mathcal{H}_{code}$$

Idea: $\tilde{N}[\rho] \stackrel{?}{=} \mathrm{Tr}_{\bar{A}} [J \rho J^\dagger]$ $\rightarrow$ recovery channel for this, bound error using JLMS

Issue: JLMS is for ρ_A ; a priori, $(J\rho J^\dagger)_A$ could depend on $\bar{a}$!

Strategy: (1) define $N[\rho_a] := \mathrm{Tr}_{\bar{A}}[J \rho_a \otimes \bar{\sigma}_a J^\dagger]$

↑ fixed full-rank fiducial

$\rightsquigarrow$ get recovery map $R_{\bar{\sigma}_a, N}$

then $-2 \log F(\rho_a, R \circ N[\rho_a]) \leq |D(\rho_a \| \sigma_a) - D(N[\rho_a] \| N[\sigma_a])| \overset{\text{JLMS}}{\leq} \varepsilon$

(2) Then, show that R still works well for arb. $\rho \in S(\mathcal{H}_{code})$

i.e. $\| \rho_a - R[(J\rho J^\dagger)_A] \|_1 \leq \delta$ * depends on ε

(3) Let $O_A := R^\dagger[\phi_a]$

$\rightsquigarrow$ show that $|\langle O_A \rangle_{J\rho J^\dagger} - \langle \phi_a \rangle_\rho| \leq \delta \| S \phi_a \|$

Notes

- cf. previous comments about factorization, J an isometry
- the fact that $\phi_a \in W_A$ entered via JLMS
- explicit formula:

let $\tau = \frac{I_{code}}{d_{code}}$ max mixed, $H_A = -\log (J \tau J^\dagger)_A$

$\rightarrow O_A = R^\dagger[\phi_a]$

$$= \frac{1}{d_{code}} \int_{\mathbb{R}} dt\, \beta_0(t)\, e^{\frac{1}{2}(1-it) H_A}\, \mathrm{Tr}_{\bar{A}}[J(\phi_a \otimes I_{\bar{a}}) J^\dagger]\, e^{\frac{1}{2}(1+it) H_A}$$